THE DERIVED CATEGORY AS AN ∞-CATEGORY

ISAIAH DAILEY

Contents

This document contains notes written for a talk given at the UCSD Winter 2024 ∞-categories seminar. The notes are closely based on the Münster lecture series on ∞-categories and higher algebra, as well as Sections 1.2 and 1.3 in [3]. Nothing here is original, and some of what is written has been copied verbatim from one of the aforementioned sources. We skip most proofs and instead give an overview of the theory, although we try to provide references for every result we do not prove.

In what follows, we fix an abelian category $\mathcal{A}$. If you're not very familiar with abelian categories, then feel free to consider the case that $\mathcal{A} = R\text{-}\mathbf{Mod}$ is the category of left R-modules for some (not-necessarily commutative) ring with identity R.

1. The construction of the homotopy ∞-category

Our goal will be to construct an ∞-categorical version of the homotopy category $K(\mathcal{A})$ of chain complexes of $\mathcal{A}$. An outline of this construction is given as follows:

1. We can define a dg-category (that is, a category enriched over the category $\mathrm{Ch}(\mathbf{Ab})$ of chain complexes of abelian groups) $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$, whose objects are chain complexes in $\mathcal{A}$.
2. The inclusion $\mathrm{Ch}_{\geq 0}(\mathbf{Ab}) \hookrightarrow \mathrm{Ch}(\mathbf{Ab})$ has a right adjoint $\tau_{\geq 0} : \mathrm{Ch}(\mathbf{Ab}) \to \mathrm{Ch}(\mathbf{Ab})_{\geq 0}$ which is lax monoidal. Thus by applying $\tau_{\geq 0}$ objectwise to the morphism objects in $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$, we can regard $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ as canonically enriched over the category $\mathrm{Ch}(\mathbf{Ab})_{\geq 0}$ of nonnegatively graded chain complexes of abelian groups.
3. The Dold-Kan correspondence supplies an equivalence of categories from $\mathrm{Ch}(\mathbf{Ab})_{\geq 0}$ to the category $\mathbf{Ab}_{\Delta}$ of simplicial abelian groups. Moreover, this equivalence is a lax monoidal functor, meaning it allows us to further view $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ as enriched in simplicial abelian groups.

Date: March 1, 2024.

4. The forgetful functor $\mathbf{Ab}_\Delta \to \mathbf{Set}_\Delta$ taking a simplicial abelian group to its underlying simplicial set is strict monoidal, thus we can further regard $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ as enriched in simplicial sets.
5. The underlying simplicial set of any simplicial abelian group is in fact a Kan complex, so we may further regard $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ as enriched in Kan complexes.
6. Finally, by applying the homotopy coherent nerve construction mentioned by Mark in his last talk (or Münster Lecture 2), we obtain an ∞-category $K(\mathcal{A}) := N_\Delta(\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A}))$.

All of the notation used above will be defined below. To start, we recall the definition of the standard 1-category of chain complexes in $\mathcal{A}$.

Definition 1.1. A *chain complex* with values in $\mathcal{A}$ is a composable sequence of morphisms (called *differentials*)

$$\cdots \to A_2 \xrightarrow{d(2)} A_1 \xrightarrow{d(1)} A_0 \xrightarrow{d(0)} A_{-1} \xrightarrow{d(-1)} A_{-2} \to \cdots$$

in $\mathcal{A}$ such that $d(n-1) \circ d(n) = 0$ for every integer n. A morphism of chain complexes $f_\bullet : A_\bullet \to B_\bullet$ is the data of arrows $f_n : A_n \to B_n$ which commute with the differentials, in the obvious sense. The collection of chain complexes with values in $\mathcal{A}$ is itself an abelian (1-)category, which we will denote by $\mathrm{Ch}(\mathcal{A})$.

For each integer n, we let $\mathrm{Ch}(\mathcal{A})_{\geq n}$ denote the full subcategory of $\mathrm{Ch}(\mathcal{A})$ spanned by those chain complexes $A_\bullet$ where $A_k \cong 0$ for $k < n$. Similarly, we let $\mathrm{Ch}(\mathcal{A})_{\leq n}$ denote the full subcategory of $\mathrm{Ch}(\mathcal{A})$ spanned by those complexes $A_\bullet$ such that $A_k \cong 0$ for $k > n$.

In the case $\mathcal{A} = R\text{-}\mathbf{Mod}$ is the category of left R-modules (for R a not-necessarily commutative ring with identity), we will simply write $\mathrm{Ch}(R)$ rather than $\mathrm{Ch}(R\text{-}\mathbf{Mod})$ ($\mathrm{Ch}_{\geq n}(R)$ and $\mathrm{Ch}_{\leq n}(R)$ are defined similarly).

Remark 1.2. In the Münster lectures on higher algebra, as well as in [3], the derived ∞-category is defined in terms of chain complexes. However, in the literature for the derived 1-category, it is more common to define the derived category in terms of *cochain complexes* (essentially the same definition, except the differentials go up rather than down in degree). Nevertheless, the theory is pretty much the same, and we will just stick with the definition given in terms of chain complexes.

Now, our goal is to turn the category $\mathrm{Ch}(\mathcal{A})$ into an ∞-category. The first step will be to make it into a dg-category.

Definition 1.3. A *differential graded category* $\mathcal{C}$ (or a *dg-category* for short) is a category enriched in $\mathrm{Ch}(\mathbf{Ab})$. We write $\mathbf{dgCat} := \mathrm{Ch}(\mathbf{Ab})\text{-}\mathbf{Cat}$ for the category of dg-categories.

In [3, Definition 1.3.1.1], Lurie explicitly unravels the gory details of this definition without mentioning monoidal or enriched catgories. For our purposes, the key example of a dg-category will be the following.

Example 1.4. Given an abelian category $\mathcal{A}$, we may form a dg-category $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ with the same objects as $\mathrm{Ch}(\mathcal{A})$.

Moreover, as a consequence of Lurie's definition of a dg-category, it is straightforward to see the following result.

Remark 1.5. Given chain complexes $M_\bullet$ and $N_\bullet$ in $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$, the m^{th} homology group $H^m(\mathrm{Hom}_{\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})}(M_\bullet, N_\bullet))$ of the associated hom chain complex is isomorphic to the group of chain homotopy classes of maps from $M_\bullet$ into the shifted chain complex $N_\bullet[m] := N_{\bullet+m}$.

An explicit construction of the hom-objects in $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ is not necessary for our purposes, although one may find this construction as [3, Definition 1.3.2.1]. It is worth noting that in [3] (and also in the Münster lectures), $\mathrm{Ch}(\mathcal{A})$ is used to refer to both the 1-category and the dg-category of chain complexes, by abuse of notation. We choose to differentiate the two.

So far, we have managed to associate a dg-category $\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})$ to each abelian category $\mathcal{A}$, whose objects are chain complexes in $\mathcal{A}$ and whose hom-objects are chain complexes of abelian groups. We would like to get an ∞-category out of this. To do so, we will prove the following result:

Theorem 1.6. *There exists a functor*

$$N_{\mathrm{dg}} : \mathbf{dgCat} \to \infty\text{-}\mathbf{Cat}\,.$$

Given a dg-category $\mathcal{C}$, we refer to the ∞-category $N_{\mathrm{dg}}(\mathcal{C})$ as the dg nerve *of $\mathcal{C}$.*

Remark 1.7. Our proof of Theorem 1.6 (which will follow the Münster lectures) will be somewhat dishonest. What we are actually going to construct is a functor which is canonically equivalent (though not isomorphic) to the usual notion of the dg nerve found in the literature ([3, Construction 1.3.16]).[1] The proof that our construction of the dg nerve is equivalent to the standard construction can be found as [3, Proposition 1.3.1.17].

The proof of Theorem 1.6 will be based on the following two results.

Remark 1.8. Given a right-lax monoidal functor $F : \mathcal{V} \to \mathcal{W}$ between monoidal categories, there is an induced *change of enrichment functor* $F_* : \mathcal{V}\text{-}\mathbf{Cat} \to \mathcal{W}\text{-}\mathbf{Cat}$ sending a $\mathcal{V}$-enriched category to a $\mathcal{W}$-enriched category. Explicitly, given a $\mathcal{V}$-enriched category C, the $\mathcal{W}$-enriched category $F_*(C)$ has the same objects as C, and the hom-objects are defined by $\mathrm{Hom}_{F_*(C)}(x, y) := F(\mathrm{Hom}_C(x, y))$.

An explicit construction of F_* on objects (which is all we will really need) can be found in [2] as Remark A.1.4.3. For a review of the definition of lax monoidal functors and enriched categories, we refer the reader to Definitions A.1.3.5 and §A.1.4 in *op. cit.*, respectively. For those unfamiliar, a functor between monoidal categories is lax monoidal if it "plays nicely" with the monoidal structures in both categories, in a sense which we will not make precise. In what follows, we will simply refer to right-lax monoidal functors as lax monoidal functors.

The second result we'll use is the following, which was introduced to us by Mark in the previous talk:

Proposition 1.9. *Let $\mathbf{Set}_\Delta\text{-}\mathbf{Cat}$ denote the category of simplicially enriched categories, i.e., the category of categories enriched over simplicial sets. Then there exists a functor*

$$N_\Delta : \mathbf{Set}_\Delta\text{-}\mathbf{Cat} \to \mathbf{Set}_\Delta$$

[1] I.e., for any dg-category $\mathcal{C}$, there will be an equivalence of ∞-categories between our construction of $N_{\mathrm{dg}}(\mathcal{C})$ and the construction of $N_{\mathrm{dg}}(\mathcal{C})$ given in [3], but this equivalence will not, in general, be an isomorphism of ∞-categories.

called the simplicial nerve taking a simplicially enriched category $\mathcal{C}$ to a simplicial set $N_\Delta(\mathcal{C})$. Moreover, given a simplicially enriched category $\mathcal{C}$, if $\mathcal{C}$ is **Kan**-*enriched (i.e., if $\mathrm{Hom}_{\mathcal{C}}(x,y)$ is a Kan complex for each pair of objects x and y in $\mathcal{C}$), then $N_\Delta(\mathcal{C})$ is an ∞-category.*

A construction of the simplicial nerve functor may be found in [2, Definition 1.1.5.5]. A proof that the simplicial nerve of a **Kan**-enriched category is an ∞-category is given in Proposition 1.1.5.10 in *op. cit.*

Thus, in order to prove Theorem 1.6, it suffices to construct a lax monoidal functor $K : \mathrm{Ch}(\mathbb{Z}) \to \mathbf{Kan}$, as then by Remark 1.8 and Proposition 1.9, we could construct the desired functor as the composition

$$N_{\mathrm{dg}} : \mathbf{dgCat} = \mathrm{Ch}(\mathbb{Z})\text{-}\mathbf{Cat} \xrightarrow{K_*} \mathbf{Kan}\text{-}\mathbf{Cat} \xrightarrow{N_\Delta} \mathbf{QuasiCat},$$

where **QuasiCat** denotes the full subcategory of $\mathbf{Set}_\Delta$ on the simplicial sets which are ∞-categories.

Now we look to construct K. To do so, we will define it as a composition of lax monoidal functors

$$\mathrm{Ch}(\mathbb{Z}) \xrightarrow{\tau_{\geq 0}} \mathrm{Ch}(\mathbb{Z})_{\geq 0} \xrightarrow{\Gamma} \mathbf{Ab}_\Delta \longrightarrow \mathbf{Kan},$$

where the first functor is a truncation functor defined in Lemma 1.10 below, the second functor is given the Dold-Kan correspondence (Theorem 1.11 below), and the final functor is given by the forgetful functor $\mathbf{Ab}_\Delta \to \mathbf{Set}_\Delta$, which factors through **Kan** by Lemma 1.13 below (here $\mathbf{Ab}_\Delta := \mathrm{Fun}(\Delta^{\mathrm{op}}, \mathbf{Ab})$ is the category of simplicial abelian groups).

First, let's define the truncation functor. There is an obvious candidate for what this functor should be: Given a complex $A_\bullet$, one might be tempted to define $\tau_{\geq 0}(A_\bullet)$ to be the complex obtained by "throwing away" (setting to zero) all the nonnegative-graded terms in the complex. This construction is called the *brutal* truncation functor, and an issue with it is that it does not preserve the homology of the chain complex in degree 0, which is a desirable property as we will later see. To remedy the situation, we instead define $\tau_{\geq 0}$ as follows.

Lemma 1.10. *There exists a lax monoidal* truncation functor $\tau_{\geq 0} : \mathrm{Ch}(\mathbb{Z}) \to \mathrm{Ch}(\mathbb{Z})_{\geq 0}$ *sending a complex $A_\bullet$ to the truncated complex $\tau_{\geq 0}(A_\bullet)_\bullet$ defined by*

$$\tau_{\geq 0}(A_\bullet)_n := \begin{cases} 0 & n < 0 \\ \ker(d(0) : A_0 \to A_{-1}) & n = 0 \\ A_n & n > 0, \end{cases}$$

with differentials defined in the obvious way.

Explicitly, this functor sends a complex

$$\cdots \to A_2 \xrightarrow{d(2)} A_1 \xrightarrow{d(1)} A_0 \xrightarrow{d(0)} A_{-1} \xrightarrow{d(-1)} A_{-2} \to \cdots$$

to the truncated complex

$$\cdots \to A_2 \xrightarrow{d(2)} A_1 \xrightarrow{d(1)} \ker d(0) \to 0 \to 0 \to \cdots$$

(where the map $d(1) : A_1 \to \ker d(0)$ is induced by the universal property of the kernel using the fact that $d(0) \circ d(1) = 0$). It is straightforward to see that $\tau_{\geq 0}$ preserves homology in nonnegative degree. Moreover, $\tau_{\geq 0}$ is in fact the right adjoint of the canonical inclusion functor $\mathrm{Ch}(\mathcal{A})_{\geq 0} \hookrightarrow \mathrm{Ch}(\mathcal{A})$. An explicit construction of the lax monoidal structure on this functor can be found in Example 3.1.6 in [4].

Theorem 1.11 (The Dold-Kan Correspondence)**.** *There exists an equivalence of categories*

$$\Gamma : \mathrm{Ch}(\mathbb{Z})_{\geq 0} \xrightarrow{\sim} \mathbf{Ab}_\Delta.$$

Moreover, this functor is lax monoidal, where $\mathbf{Ab}_\Delta := \mathrm{Fun}(\Delta^{\mathrm{op}}, \mathbf{Ab})$ *is the category of simplicial abelian groups equipped with the monoidal structure given by degree-wise tensor product.*

Example 1.12. Given an abelian group A and a nonnegative integer n, we may view A as an object in $\mathrm{Ch}(\mathbb{Z})_{\geq 0}$ as the chain complex containing A in degree n and zero in all other degrees. Then $\Gamma(A)$ is the Eilenberg-MacLane space $K(A, n)$.

The reader may refer to [3] for the proof of Theorem 1.11. There the functor is constructed (in slightly more generality) in Construction 1.2.3.5 (where it is referred to as DK rather than Γ). The functor Γ is proven to be an equivalence in Theorem 1.2.3.13, and Example 1.2.3.26 explains why it is lax monoidal. Alternatively, [4, Lemma 3.2.5] gives a more fleshed-out proof that this functor is lax monoidal.

Lemma 1.13. *The underlying simplicial set of a simplicial abelian group is a Kan complex. I.e., the forgetful functor* $\mathbf{Ab}_\Delta \to \mathbf{Set}_\Delta$ *factors as a functor* $\mathbf{Ab}_\Delta \to \mathbf{Kan}$*. Moreover, this functor is lax monoidal (with respect to the monoidal product on* $\mathbf{Ab}_\Delta$ *given by degree-wise tensor product and the cartesian monoidal structure on* $\mathbf{Kan}$*).*

A proof that the underlying simplicial set of any simplicial group is a Kan complex is given as Lemma 3.4 in [1], or as Corollary 1.3.2.12 in [3]. It is straightforward to see it is lax monoidal.[2]

Thus, we have constructed the functor $K : \mathrm{Ch}(\mathbb{Z}) \to \mathbf{Kan}$ sending a chain complex of abelian groups $A_\bullet$ to the underlying simplicial set of the simplicial abelian group $\Gamma(\tau_{\geq 0} A_\bullet)$. Thus, the homotopy ∞-category of chain complexes of $\mathcal{A}$ can be defined to be the simplicial nerve of the category $K_*(\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A}))$, which may be described more concretely as follows:

- The objects of $K_*(\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A}))$ are chain complexes in $\mathcal{A}$.
- Given chain complexes $A_\bullet$ and $B_\bullet$ in $\mathrm{Ch}(\mathcal{A})$, the hom object

  $$\mathrm{Hom}_{K_*(\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A}))}(A_\bullet, B_\bullet)$$

  is the underlying simplicial set of the simplicial abelian group

  $$\Gamma(\tau_{\geq 0} \mathrm{Hom}_{\mathrm{Ch}_{\mathrm{dg}}(\mathcal{A})}(A_\bullet, B_\bullet)).$$

2. Dwyer-Kan Localization of ∞-categories and the construction of the derived ∞-category

Definition 2.1. Let $\mathcal{C}$ be an ∞-category, and suppose $\mathcal{W}$ is a collection of morphisms in $\mathcal{C}_1$. Then a functor $F : \mathcal{C} \to \mathcal{C}'$ between ∞-categories is a *Dwyer-Kan localization at* $\mathcal{W}$ if

- The functor F takes every morphism in $\mathcal{W}$ to an equivalence in $\mathcal{C}'$,

[2] Explicitly, given simplicial abelian groups A and B, we need a map $A \times B \to A \otimes B$. There is an obvious one, sending $(a, b) \mapsto a \otimes b$. Checking these maps satisfy the required coherence conditions is straightforward.

- For any category $\mathcal{D}$, the pushforward $\mathrm{Fun}(\mathcal{C}', \mathcal{D}) \to \mathrm{Fun}^{\mathcal{W}}(\mathcal{C}, \mathcal{D})$ is an equivalence of ∞-categories, where $\mathrm{Fun}^{\mathcal{W}}(\mathcal{C}, \mathcal{D}) \subseteq \mathrm{Fun}(\mathcal{C}, \mathcal{D})$ is the full ∞-subcategory on those functors $\mathcal{C} \to \mathcal{D}$ which take morphisms in $\mathcal{W}$ to equivalences.

In this case, we write $\mathcal{C}[\mathcal{W}^{-1}] = \mathcal{C}'$.

Example 2.2. Let $\mathcal{C} = \Delta^1$, and let $\mathcal{W} \subseteq \mathcal{C}_1$ contain only the single nontrivial morphism $0 \to 1$ in Δ^1_1. Then $\mathcal{C}[\mathcal{W}^{-1}]$ is equivalent to Δ^0.

Proof sketch. We wish to show the unique functor $F : \Delta^1 \to \Delta^0$ induces an equivalence of ∞-categories $F^* : \mathrm{Fun}(\Delta^0, \mathcal{D}) \to \mathrm{Fun}^{\mathcal{W}}(\Delta^1, \mathcal{D})$ for all ∞-categories $\mathcal{D}$.

First of all, in order to see it is essentially surjective, note an object in $\mathrm{Fun}^{\mathcal{W}}(\Delta^1, \mathcal{D})$ is precisely the data of an equivalence $f : d \xrightarrow{\sim} d'$ in $\mathcal{D}$. Moreover, $\mathrm{Fun}(\Delta^0, \mathcal{D})$ is clearly isomorphic to $\mathcal{D}$, and the functor $F^* : \mathcal{D} \to \mathrm{Fun}^{\mathcal{W}}(\Delta^1, \mathcal{D})$ sends an object d in $\mathcal{D}$ to the identity morphism $\mathrm{id}_d = s_0 d : d \to d$. Thus, this problem can be reduced to showing that every object $(d \xrightarrow{\sim} d') \in \mathrm{Fun}^{\mathcal{W}}(\Delta^1, \mathcal{D})$ is equivalent to $(\mathrm{id}_d : d \to d)$, which is a straightforward exercise in simplicial sets.

To see fully faithfulness (that F^* induces an equivalence of mapping spaces), first one shows that given equivalences $f : a \xrightarrow{\sim} a'$ and $g : b \xrightarrow{\sim} b'$ in $\mathcal{D}$, the following diagram is a pullback diagram in the ∞-category $\mathcal{S}$ of spaces.

$$\begin{array}{ccc} \mathrm{Map}_{\mathrm{Fun}^{\mathcal{W}}(\Delta^1, \mathcal{D})}(f, g) & \longrightarrow & \mathrm{Map}_{\mathcal{D}}(a', b') \\ {\scriptstyle F^*}\big\downarrow & \lrcorner & \big\downarrow{\scriptstyle f^*} \\ \mathrm{Map}_{\mathcal{D}}(a, b) & \xrightarrow{g_*} & \mathrm{Map}_{\mathcal{D}}(a, b') \end{array}$$

We know the right arrow is an equivalence because f is, and equivalences are preserved under pullbacks, so the left arrow is an equivalence as well, which gives the desired result. □

Proposition 2.3. *Dwyer-Kan localizations always exist.*

Proof. Let $\mathcal{C}$ and $\mathcal{W}$ as in Definition 2.1, and construct $\mathcal{C}'$ as the following pushout in $\mathbf{Cat}_\infty$.

$$\begin{array}{ccc} \coprod_{\mathcal{W}} \Delta^1 & \longrightarrow & \mathcal{C} \\ \big\downarrow & \ulcorner & \big\downarrow \\ \coprod_{\mathcal{W}} \Delta^0 & \longrightarrow & \mathcal{C}' \end{array}$$

where the top arrow picks out $\mathcal{W}$, and the right arrow is a $\mathcal{W}$-indexed coproduct of copies of the unique $\Delta_1 \to \Delta_0$. Then one proves that given any ∞-category $\mathcal{D}$, applying $\mathrm{Fun}(-, \mathcal{D})$ to the diagram and using the universal property of the coproduct yields a pullback diagram of spaces

$$\begin{array}{ccc} \mathrm{Fun}(\mathcal{C}', \mathcal{D}) & \longrightarrow & \prod_{\mathcal{W}} \mathrm{Fun}(\Delta^0, \mathcal{D}) \\ \big\downarrow & \lrcorner & \big\downarrow \\ \mathrm{Fun}(\mathcal{C}, \mathcal{D}) & \longrightarrow & \prod_{\mathcal{W}} \mathrm{Fun}(\Delta^1, \mathcal{D}) \end{array}$$

Then by Example 2.2 we know that the right arrow is an equivalence, so the left arrow is as well, as desired. □

Finally, with this we may define the derived ∞-category $\mathcal{D}(\mathcal{A})$ of $\mathcal{A}$. as the Dwyer-Kan localization of

Definition 2.4. Given an abelian category $\mathcal{A}$, we define the *derived* ∞*-category of* $\mathcal{A}$ as

$$\mathcal{D}(\mathcal{A}) := \mathcal{K}(\mathcal{A})[\mathcal{Q}^{-1}],$$

where $\mathcal{Q}$ is the collection of *quasi-isomorphisms* in $\mathcal{K}(\mathcal{A})$, i.e., those morphisms of chain complexes which induce isomorphisms on homology.

References

[1] Goerss, P.G., Jardine, J.F.: Simplicial homotopy theory, *Prog. Math.*, vol. 174. Basel: Birkhäuser (1999)

[2] Lurie, J.: Higher topos theory, *Ann. Math. Stud.*, vol. 170. Princeton, NJ: Princeton University Press (2009). DOI 10.1515/9781400830558

[3] Lurie, J.: Higher algebra (2017). URL https://people.math.harvard.edu/~lurie/papers/HA.pdf. Unpublished.

[4] Opadotun, M.M.: Simplicial enrichment of chain complexes. Master's thesis (2021). URL https://ourspace.uregina.ca/items/73614acf-77c6-4e43-aee4-91f16571b704

Email address: isaiah@ucsd.edu